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A simultaneous equation model to determine the rate of inflation

Mary Elizabeth Rieder
Iowa State University

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A simultaneous equation model to determine the

rate of inflation

by

Mary Elizabeth Rieder

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DEDICATION

To "Chris"

Who took the time and had
the patience to make me an
economist

I. INTRODUCTION

In contrast with earlier periods in the development of economic theory, the basic problem now facing economists is not one of insufficient aggregate demand and unemployment, but inflation. Inflation is generally considered to be a problem associated with full employment and excess aggregate demand, rather than with unemployment and insufficient aggregate demand.

Inflation is also essentially a monetary rather than a real phenomenon. The Classical economists clearly realized this. Postulating the exchange identity, $MV = PY$, where M is the money supply, V is income velocity, P is the price level, and Y is the level of real output, they argued if the economy is at full employment and V is constant, prices would rise in proportion to increases in the money supply. Even if Y and V are not held constant, but are allowed to vary within reasonable limits consistent with the concept of full employment and the institutional framework of the economy, the relationship between the money supply and the price level, while no longer proportional, is still very strong.

It would seem that the money supply would be the key variable in determining the price level in macroeconometric models. However, in the major quarterly econometric models

of the United States, this is not the case. David Fand
(40) points out that the price level in many of these current
econometric models depends on such variables as wages, unit
labor costs, variable markup, capacity utilization, unfilled
orders, shipments, and farm and import prices. These
variables take into account changes in costs, shifts in
demand, productivity trends, tax changes etc., but do not
explicitly take into account the effect on prices of changes
in the money supply. Fand argues these models use basically
the microeconomic approach of supply and demand analysis
to explain a macroeconomic price variable. This approach
poses two problems. First, movements in an aggregate index
cannot be adequately explained using a theory of relative
prices. And second "...this demand-supply theory suggests
that changes in the cost of production (an autonomous rise
in the money wage rates, in the markup factor, or in
profits) will cause general commodity prices to rise without
specification of the monetary requirements necessary to
generate rising prices or validate the higher price level"
(Fand 40, p. 451). The current macroeconometric models do
not have a monetary theory of the price level.

In these models the main reason for the non-monetary

approach to inflation lies in their basic structure. These models are built in the Keynesian tradition where the problems of recession, insufficient demand, and unemployment are of primary interest. Assuming that aggregate supply will rise to meet aggregate demand, the emphasis in these models is on aggregate demand and its components: consumption, investment, and government spending, if a closed economy is assumed. In order to highlight aggregate demand, these models subdivide into a consumption sector, an investment sector, and a fiscal sector. The level of output and the level of employment depend on the level of aggregate demand as determined in the various sub-sectors of the model. Changes in policy variables are translated first into changes in demand, thus indirectly affecting the level of output and employment.

To determine the effect of changes in the money supply, for example, on the level of prices, output, and employment, changes in the money supply must first be translated into changes in consumption, investment, or government spending. Typically changes in the money supply are translated into changes in demand through changes in the interest rate. The interest rate mechanism operates in the manner to be described. The money supply change causes interest rates to change. The interest rate changes will lead to changes in consumption and investment. For example, the FRB-MIT-Penn

model postulates first a change in the money supply which affects interest rates. The changes in the interest rates change the various investment variables through the cost of capital variables. Consumption is directly influenced by changes in interest rates and is also affected by the interest rate effect on wealth. Housing expenditures are directly affected by interest rate changes as well as credit rationing variables, (25). Autonomous changes in the money supply in general will not affect the level of government spending.[1]

The effect of changes in the money supply on prices, output and employment is obscured in these models, because the effect of monetary changes is not direct, but must operate through the interest rate mechanism. The interest rate is, however, a price. And, like any price, it is determined by the interplay of the forces of supply and demand. A

[1]The converse, however, is not necessarily true. If there is an increase in government spending, government can obtain its spending power in three ways. The first is to increase taxes, causing declines in consumption and investment to offset the increase in government spending. The second is deficit financing through the issue and sale of bonds to the public. The public trades present consumption and investment for future consumption and investment, freeing current goods and services for government use. The third is deficit financing through the issue and sale of government bonds to the Federal Reserve. This increases the money supply to the extent that the Federal Reserve does not offset this purchase of bonds by open market operations. Therefore, although the autonomous shifts in the money supply do not affect the level of government spending, the method used to finance government spending may change the money supply.

change in supply or demand will affect the interest rate. We
can isolate the effect of monetary actions on the interest
rate only if we hold demand constant as well as the other
factors that influence supply. And this is impossible in a
model built to explain actual economic phenomena. The full
implications of using an interest rate to reflect monetary
actions is discussed more fully by Starleaf and Stephenson
(107), and in other literature on the monetary indicator
problem (14, 15).

Further, in an inflationary period the money supply
effect in changing interest rates becomes impossible to iso-
late. The reason for this is seen in Irving Fisher's distinc-
tion between real and nominal rates of interest. The nominal
or market rate of interest measures the rate of exchange
between present and future dollars. The real rate of interest
measures the rate of exchange between present and future
goods and services. The difference between them is the ex-
pected rate of inflation. In equation form: $Rn = Rr + E(\dot{P})$,
where Rn is the nominal rate of interest, Rr is the real
rate, and $E(\dot{P})$ is the expected rate of inflation. Clearly
if $E(\dot{P})$ is zero, the nominal and real interest rates are
identical. But if the rate of price increase is not zero,
nominal interest rates, Rn, will change as inflationary
expectations change. Hypothetically, during an inflationary
period because of expected continued inflation, nominal

interest rates could rise although the money supply was also increasing during the same period. If a monetary rather than an inflationary interpretation is made of the increasing interest rates, the conclusion would be that the money supply is declining and should perhaps be increased to keep interest rates from rising even further.

The interest rate mechanism gives potentially false information about the direction and impact of money supply changes. This is especially true in an inflation. It is, therefore, an unacceptable mechanism in models developed to explain the inflationary process in monetary terms.

It is the purpose of this dissertation to formulate an econometric macroeconomic model that will assess <u>directly</u> the effect of changes in the money supply on prices as well as output and employment.

Once this is accomplished, it should be possible in such a model to discern a rate of growth of the money supply consistent with the widely sought economic goals of either price stability, full employment, or economic growth.

Much has been written on an optimum rate of growth in the money supply, particularly within the context of growth models. Milton Friedman (48) has recently estimated that the optimum rate of growth of the money stock for price stability in the U.S. is about 2%. His estimate is not derived from a fully specified interdependent system of equa-

tions estimated simultaneously, but from single equation
reduced form models of the demand for money.

It is the second objective of this dissertation to
evaluate this suggested rate of monetary growth within the
context of a simultaneous system of equations and to deter-
mine if the result generated from such a system of equa-
tions is consistent with that suggested by Dr. Friedman.

II. THE MODEL AND THE ESTIMATION OF THE
STRUCTURAL EQUATIONS

A. The Model

In order to develop a model that directly accounts for
changes in the money supply on prices, we begin by looking
first at the formulation of a traditional Keynesian model.
In such a model, aggregate demand for goods and services is
defined as the sum of consumption, gross investment, and
government spending on goods and services, assuming a closed
economy. As an identity this can be expressed as the familiar
$PY = C + I + G$, where PY is nominal aggregate demand broken
down into its constituent parts: C, consumption; I gross
investment; and G, government spending. Making the basic
Keynesian assumption that aggregate supply will rise to meet
aggregate demand, the definition becomes a statement of supply
and demand equilibrium. If aggregate demand in real terms
just equals aggregate supply in real terms, the economy
will be in equilibrium. If aggregate demand is greater
than aggregate supply, disequilibrium exists, setting into
motion forces in the economy that will return the economy
to equilibrium by increasing either prices and/or output.

This same aggregate supply and demand relationship can
also be constructed from the quantity theory of money. In
terms of this theory, over a specified time period, the
supply of money times income velocity must equal the average

price of goods and services sold during the period multi-
plied by the amount of real output sold. In equation form
this relationship is defined by the familiar equation of
exchange: $MV = PY$. PY is aggregate supply, while the
interaction of M and V can be interpreted to represent aggre-
gate demand. Actual velocity times the money supply must
equal the nominal value of the goods and services sold. In
this formulation, the equation is an identity.

If <u>desired</u> money balances are substituted for the money
supply, the equation then represents an equilibrium condition
which states that aggregate demand and aggregate supply
will be equal if, and only if, actual money balances are
equal to desired money balances. If the actual and desired
values deviate, it is possible to have either excess demand
or supply in the aggregate economy. If the supply of money
is greater than the amount of money that people desire to
hold, it is assumed that people attempt to spend away their
excess money balances. It follows that either C or I or G
has to increase. This increase in aggregate demand has the
effect of calling forth increases in either prices or real
output or a combination of the two, depending upon the
availability of unutilized productive capacity.

Employing the above analysis, the model we have speci-
fied is developed in two sectors. One sector is formulated
to represent aggregate demand. The other sector is formu-

lated to embody the forces pertinent to aggregate supply. Price increases, or inflation, are then viewed within the context of the model as resulting from the difference between aggregate supply and demand.

The model consists of eight equations: a price equation, a velocity equation, an output equation, an employment equation, and a wage equation, plus three definitional identities to close the model. The model is specified in log-linear form since four of the five basic equations: prices, velocity, output, and employment are theoretically multiplicative, and the wage equation is multiplicative in the demand for labor and no a priori reason exists to reject a multiplicative formulation for the supply of labor as well.

The estimates of each of the structural equations are based on quarterly observations from 1952 to 1969. Each of the equations is estimated by ordinary least squares or generalized least squares. Two stage least squares estimation was attempted, but in terms of the predictive accuracy in the reduced form of the model, the ordinary least squares estimates proved to be superior.

The Model:

The model consists of eight equations, specified as follows:

(1) $\ln P_t = \ln M_t + \ln V_t - \ln Y_t$

(2) $\ln V_t = \alpha_{21} \ln \beta_o + \alpha_{22} \ln Y_t + \alpha_{23} \ln P_t - \alpha_{24} \ln P_{t-1}$

(3) $\ln [Y/E]_t = \ln A_t + \beta_1 \ln [K/E]_t$

(4) $\ln E_t = \alpha_{41} \ln \beta_2 - \alpha_{42} \ln A_t + \alpha_{43} \ln Y_t - \alpha_{44} \ln [W/E]_t$

$\qquad + \alpha_{45} \ln P_t$

(5) $\ln W_t = \alpha_{51} \ln \beta_2 + \alpha_{52} \ln Y_t + \alpha_{53} \ln P_t + \alpha_{54} \ln N_t$

$\qquad + \alpha_{55} \ln E_t$

(6) $\ln Y_t = \ln [Y/E]_t + \ln E_t$

(7) $\ln [W/E]_t = \ln W_t - \ln E_t$

(8) $\ln [K/E]_t = \ln K_t - \ln E_t$

Where P is the price level, P_{t-1} is the price level lagged 1 quarter; M is the money supply; V is velocity; Y is real income; E is the number of men and women employed; Y/E is real income divided by the number employed; A is the level of technology; K is the capital stock; K/E is the capital stock divided by employment; and N is the size of the population between the ages of 14 and 64. All variables are measured in the current period, except for P_{t-1}, which is prices lagged one quarter. The α's represent the coefficients on the variables. If there is no α, the coefficient is one. The source of the data used in the estimation of the model can be found in Appendix A.

B. The Price Equation

The price equation is quickly recognizable as a simple
variant of the equation of exchange, MV = PY. To derive
this equation, the natural log is taken of the equation of
exchange and the price variable is algebraically isolated:

(1) $\ln P_t = \ln M_t + \ln V_t - \ln Y_t$

Prices change when there is disequilibrium between
aggregate demand and aggregate supply. Aggregate demand in
the model is embodied in ln M + ln V. Aggregate supply is
represented by ln Y. In this model, it is clear that the
money supply itself is the crucial variable. As the money
supply and/or velocity increase relative to aggregate out-
put, prices will rise.

As discussed in the introduction, we do not employ an
interest rate to determine the effect of monetary policy on
the macro level of prices, wages, employment, and output.
In this model we are able to obtain the direct effect of
monetary changes. This is desirable for three reasons. First,
we are interested in the use of the money supply rather than
the interest rate as the monetary policy variable. Second,
the interest rate in a simultaneous equation system should
really be an endogenous rather than an exogenous variable
if it is used, and at this point in the development of the

model we are not ready to include a financial sector to determine interest rates. Finally, the interest rate in an inflation model is a difficult variable to interpret and link directly with changes in the money supply because of the strong relationship between the rate of inflation and the nominal interest rate.

Changes in demand are seen in the price equation through the direct interaction of changes in the money supply and changes in velocity. In this formulation of the model the money supply is exogenous, while velocity and output are endogenously determined in later equations.

C. The Demand Sector

Because of the exogeneity of the money supply, the only behavioral equation on the demand side of this model is the velocity equation. The velocity equation is derived in the following manner. We begin again with the equation of exchange:

(2a) $MV = PY$

and velocity is isolated by dividing both sides by M:

(2b) $V = \dfrac{PY}{M}$

then the numerator and denominator are both divided by P:

(2c) $V = Y/M/P$

In this form of the equation, it is easily seen that velocity
is defined as the ratio of real income to real money balances.
To make it a behavioral equation we specify that desired
velocity is the ratio of real income to desired money
balances. The literature of the demand for money typically
specifies the following money demand relationship:

(2d) $M/P = \beta_0 \, Y^\delta r^{-x}$

where the demand for money balances depends positively on the
level of income representing a transactions balance effect,
and negatively on the interest rate representing the oppor-
tunity cost effect.

It can be argued that the rate of change of prices is
superior to the interest rate as a determinant of money
demand for the rate of change of prices picks up two different
elements of opportunity cost. First, there should be a de-
crease in desired money balances as a response to inflation
itself. Inflation erodes the purchasing power of idle money
balances, and, in fact, acts as a tax on money balances.
Second, the rate of inflation picks up the opportunity cost
element embodied in the rate of interest. Given the rela-
tionship between the market rate of interest and the rate of
inflation discussed in the introduction it follows that the
higher the rate of inflation, the higher the nominal rate of

interest, and the higher the opportunity cost of holding money.

We replace the interest rate in the money demand function with an argument for the rate of inflation:

$$(2e) \quad M_t/P_t = \beta_o \, Y_t^{\delta_1} \, (P_t/P_{t-1})^{-\delta_2}$$

The negative coefficient, $-\delta_2$, can be rationalized along the same lines as the negative coefficient for the rate of interest. The higher the opportunity cost of holding money balances, the lower the money balances held, yielding, the negative coefficient.

The velocity function is derived by substituting Equation (2e) for the denominator of (2c):

$$(2f) \quad V = Y_t/\beta_o \, Y_t^{\delta_1} (P_t/P_{t-1})^{-\delta_2}$$

Taking the natural log of both sides, we obtain:

$$(2g) \quad \ln V_t = -\ln \beta_o + (1-\delta_1)\ln Y_t + \delta_2 \ln P_t - \delta_2 \ln P_{t-1}$$

The empirical estimate of this equation, based on the period 1952 to 1969, is as follows: ("t" values are in parentheses)

$$\ln V_t = \underset{(5.56)}{-1.9385} + \underset{(9.131)}{.5139} \ln Y_t - \underset{(3.898)}{3.106} \ln P_t + \underset{(4.878)}{3.817} \ln P_{t-1}$$

$$\overline{R}^2 = .9829$$

$$DW = .3998$$

The sign and size of the coefficient on real income is consistent with theoretical expectations. Disentangling the coefficient on Y to obtain δ_1, the income elasticity of money demand, yields a value of .4861. This estimate is consistent with those of other studies relating real money balances, measured as demand deposits plus currency, to real income and interest rates. For example, Dickson and Starleaf in their forthcoming paper (30) relate the demand for real money balances to real income, a market rate of interest, the yield on commercial bank time deposits, and the aggregate price index. They estimated the equation with the Almon lag structure, and report a coefficient on ln y* of .526.

The reversed signs on prices is puzzling. The net effect is, negative $(-3.106 \ (P_t - P_{t-1}) + .7 \ P_{t-1})$ which is opposite of what the theory would lead us to expect.

The low Durbin-Watson is, of course, of concern. An attempt was made to estimate this equation by generalized least squares. The generalized least squares (GLS) was performed by using a first order auto-regressive scheme on the residuals from the OLS equation. The original data were transformed on the basis of the parameter from the residual regression. The function was then re-estimated using the transformed variables in place of the original variables. The transformed variables took the form of $Y_t' = Y_t - \rho Y_{t-1}$. The

first "pass" using GLS resulted in the loss of significance on P_t, unrealistic regression coefficients, and an even lower Durbin-Watson. The equation was subjected to a second round of GLS and similar results were obtained in terms of significance and coefficient size, but the Durbin-Watson moved from positive serial correlation to negative serial correlation. We decided to acknowledge the presence of serial correlation in the estimated equation and proceed.

D. The Supply Sector

1. The production function

The supply sector is dominated by the aggregate production function. Following Valavanis (113), the aggregate supply function employed is the well known and widely used Cobb-Douglas production function. For our model we require that the Cobb-Douglas be homogenous of degree one, that is $\beta_1 + \beta_2 = 1$:

$$(3a) \quad Y = AK^{\beta_1} E^{\beta_2}$$

Ideally we would like to have K represent the flow of capital services. We would also like an employment measure purged of underemployment and overtime qualifications. It would also be most desirable to have an independent measure of A, like the one constructed by Solow (104) using Goldsmith's data for the period 1909 to 1943. In all cases, data limitations forced the use of less than ideal measures

of these variables. Capital is entered as a stock. A, the level of technology, is proxied by a measure of output per man hour. And E, employment, is the number of persons employed.

There are several different ways of expressing a Cobb-Douglas production function for estimation. The one we chose was the one that performed the best, that is, the one that gave the most consistent predictions in the reduced form of the model. This form of the Cobb-Douglas is derived as follows: First divide both sides of (3a) by E:

(3b) $Y/E = AK^{\beta_1} E^{\beta_2-1}$

By the assumption of 1st degree homogeneity, $\beta_2-1 = \beta_1$, and (3b) becomes:

(3c) $Y/E = A(K/E)^{\beta_1}$

Taking the natural logs, we obtain:

(3d) $\ln Y/E = \ln A + \beta_1 \ln K/E$

The estimated equation is as follows:

$\ln Y/E = .6617 \ln A + .832 \ln K/E$ $\bar{R}^2 = .9997$
       (6.972)   (58.233) $DW = .946$
                                            $\rho = .93$

It is noted that despite using generalized least squares, the Durbin-Watson remains low. It is also noted that the

coefficient on K/E is rather high for the exponent for the

capital input in production, although it is of the same

order of magnitude (.7) obtained by Valavanis[2] (113).

The capital stock in this formulation of the model is

taken as exogenous, even though it logically should be

endogenous. To endogenize the capital stock would require

the specification of an investment function as well as an

accumulation function. This would involve the introduction

of a series of linear equations and identities yielding a

mixed system of linear and nonlinear equations. While pro-

cedures are currently available for handling mixed systems

of equations, the model would have become substantially more

complicated. It was decided at this stage in the develop-

ment of the model to enter the capital stock as an exogenous

variable.

---

[2]Because of our concern over the size of this coefficient, a different form of the equation was estimated. This formulation took the form of:

(3e) $\ln Y/E = con + \beta_1 \ln K/E$

and the estimate of this equation is:

(3f) $\ln Y/E = 5.753 + .401 \ln K/E \quad \bar{R}^2 = .999$
       (2.8)   (2.2)   DW = 1.066
                                         $\rho = .949$

β_1 in this form of the equation is equal to .401, which would seem more reasonable and is also close to Solow's (104) esti-mate of .353 for annual data from 1909-1943. This form of the equation when used in place of (3d) in the simultaneous model, proved inferior in generating the reduced form pre-dictions, so the presented equation (3d) was chosen.

2. The employment function

There has existed for quite some time a debate in the literature over the role of the real wage in the economy. The marginal productivity theory of distribution assigns to the real wage the role of clearing the labor markets. By contrast, the Cambridge school[3] argues that the main function of the real wage is to create enough income for workers to clear the commodity market. The marginal productivity argument focuses on aggregate supply, while the Cambridge school emphasizes the demand side.

Solow and Stiglitz (105) have constructed a theoretical model which incorporates both these theories of the real wage. Their main arguments will be briefly summarized here because we draw on them in the development of the employment equation.

Their model first specifies an aggregate production function. In the short run, aggregate supply is postulated as a function of the labor input only: $Y^S = F(N)$, $F'(N) > 0$, $F''(N) < 0$. Under more or less competitive conditions, aggregate supply will be near the profit maximizing level of output where price is equal to marginal cost. Because labor is the only input, this is tantamount to arguing that to be on the aggregate supply curve labor will be hired up to

---

[3] So named, because this explanation of the role of the real wage has been discussed at various times by Nicholas, Kaldor, Joan Robinson, and Luigi Pasinetti.

the point where the real wage rate paid equals the marginal product of labor. Further, because of decreasing marginal productivity, aggregate output can be seen as an inverse function of the real wage. Thus, $Y^s = F^{-1} (W/P) = G(W/P)$, where $G' < 0$.

On the demand side, aggregate demand is seen as an increasing or constant function of the real wage. The higher the real wage, the more purchasing power wage earners have. Aggregate demand is specified as: $Y^d = H(W/P)$, $H' > 0$. There then exists some real wage, W/P^*, such that aggregate supply and aggregate demand are equal. If the actual real wage is below $(W/P)^*$, aggregate supply will be greater than aggregate demand. In this case actual output will be limited to the level of aggregate demand. Producers will produce what they can sell. When the level of output is determined by the level of aggregate demand, which is less than the desired level of aggregate supply at the going real wage, producers are forced to move away from their profit maximizing level of output and employment. The marginal product of labor employed is greater than the real wage, or price is greater than marginal cost. It would seem that each competitive producer would increase his profits by increasing output and employment, but all producers together can sell no more than Y^d, the level of aggregate demand. Therefore, excess supply is incompatible with perfect compe-

tition. Arrow (2), suggests that a way to solve this theoreti-
cal problem is to postulate that when there is excess supply,
markets become imperfectly competitive. Each producer now
views himself as facing a falling, rather than a horizontal
demand curve for his products. Hence, if aggregate demand
is less than aggregate supply at the going real wage, the
level of aggregate demand dominates the production decisions
of suppliers who remain off their desired supply schedules
until demand rises to the equilibrium level of output. When
the real wage is greater than $(W/P)^*$, there is excess demand
in the economy and producers will produce along their aggre-
gate supply function.

In terms of an illustration:
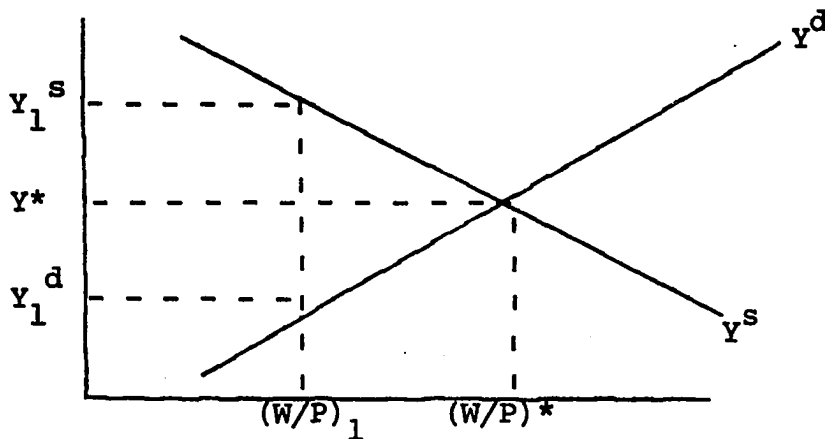
Aggregate demand, Y^d, is seen as a positive function of
the real wage, (W/P). Aggregate supply, because of diminish-
ing marginal returns to labor, exhibits a negative slope.
$(W/P)^*$ and Y^* are that real wage and output level required
to equate aggregate demand and supply. The relevant portion

of the graph is the inverted "v", whose zenith is Y*. Using
this illustration, it can be seen that if the real wage lies
to the left of $(W/P)*$, say $(W/P)_1$, producers wish to supply
Y_1^s, which is greater than the amount of aggregate demand,
Y_1^d, generated at the real wage $(W/P)_1$. In this case, the
level of aggregate demand prevails and actual output equals
Y_1^d. Anywhere on the left leg of the inverted "v", producers
are producing at an output level below the optimal level.
At this level of production, price is greater than marginal
cost, or the marginal product of labor is greater than the
real wage. As the real wage rises toward $(W/P)*$, output will
increase in response to the increase in demand which is the
result of the increase in the real wage. This allows
producers to move toward their desired level of output which
is represented by the supply curve.

Once the real wage rises to $(W/P)*$, the argument be-
comes asymetrical as output no longer responds to changes in
demand, but is now controlled by the conditions affecting
the aggregate supply function. As the real wage is forced
above the equilibrium level, to the right of $(W/P)*$,
producers will now produce along the downsloping right leg
of the inverted "v", which is the relevant portion of the
aggregate supply function. Every point on this curve
represents a price equal to the marginal cost condition. As
the real wage continues to move to the right of $(W/P)*$,

output and employment will <u>decrease</u>, leaving the economy in a period of excess demand. To require that production be equal to aggregate demand at a real wage greater than (W/P)* would force production to an output level where price was less than marginal cost causing profits to fall. This is an untenable position in a free enterprise economy where each producer makes his own production decisions. A producer finding himself at a level of production where profits are falling would either reduce production by laying off workers until he was back at his profit maximizing level of production, given the real wage, or he would raise his price causing the real wage to fall back toward equilibrium.

The implications of this analysis for employment are interesting. Because $Y = F(N)$, it can be seen that as the economy moves from $(W/P)_1$ toward equilibrium at $(W/P)*$, the effect of the higher wage rate will increase aggregate demand, and employment will increase as output rises to meet the increase in demand. In this situation, employment can be seen as an increasing function of the real wage. This finding helps explain the phenomenon of increasing real wages and increasing employment at the beginning of the business cycle. This positive functional relationship continues until $(W/P)*$ is reached. As mentioned previously, the argument becomes asymetrical at this point. If the real wage is pushed beyond $(W/P)*$, employment will <u>fall</u> as producers move

down along the aggregate supply curve. In this portion of
the inverted "v", employment is seen as an inverse function
of the real wage.

From this model a possible explanation of a period
characterized by both inflation and unemployment can be
attempted. Given large wage increases during a boom, the
real wage will be pushed beyond (W/P)* if prices do not keep
pace. Because the new real wage is now greater than the
equilibrium real wage, employment and output will fall as
producers adjust their output to a level where the marginal
physical product of labor is equal to the real wage. At the
same time high wages for those who remain working keep
aggregate demand from falling in proportion to employment
cuts. Hence, inflation occurs as aggregate demand exceeds
aggregate supply, although there is unemployment in the
economy. If wages are held fairly constant as prices rise
in this situation, the real wage will fall and the economy
will move back toward equilibrium at the lower equilibrium
wage with higher output and employment.

It should be noted here that this analysis does not
insure full employment at equilibrium. It is not necessary
in this formulation for (W/P)* to be equal to the real wage
that will clear the labor market. An equality between
aggregate supply and demand could occur at a level of output
requiring less than full employment of the labor force.

With this model of the relationship between employment and the real wage, we derive the employment function used in our model. We begin with the specification of the limiting case of the marginal productivity determination of the real wage-employment relationship. We will then modify the function by incorporating a correction factor for periods when employment depends not on the equality of productivity and the real wage, but on the level of aggregate demand. When aggregate demand dominates production decisions, the real wage will be less than the marginal product of labor.

First the aggregate production function is again postulated. Because this model is to be estimated from 1952 to 1969 it would be inconsistent to specify a short run production function where output is a function of labor only. What immediately follows is essentially a short run argument, but the function is corrected to incorporate long term elements later in the development.

Again we postulate the Cobb-Douglas production function:

$$(4a) \quad Y = AK^{\beta_1} E^{\beta_2}$$

and taking the first derivative with respect to employment:

$$(4b) \quad \partial Y/\partial E = A\beta_2 K^{\beta_1} E^{\beta_2 - 1}$$

which can be rewritten as:

(4c) $\partial Y/\partial E = \beta_2 (AK^{-\beta_1} E^{\beta_2}) E^{-1}$

Noting the expression in parentheses is equal to Y, we substitute and obtain the following:

(4d) $\partial Y/\partial E = \beta_2 Y/E$

Then assuming more or less competitive conditions, we can assume that to be on the profit maximizing aggregate supply curve labor will be paid a real wage rate, (W/EP)[4], equal to its marginal product:

(4e) $\partial Y/\partial E = (W/EP)^{\phi_1}$

If the equality holds exactly, such that we are always on the aggregate supply curve, price will be equal to marginal cost and ϕ_1 will equal 1, and should estimate so empirically. We anticipate that during certain quarters over which the function is estimated, we will not be on the aggregate supply function. We allow for this by using the power of ϕ_1 on (W/EP), and allowing for a less than unity coefficient empirically. It should be noted that we would not expect ϕ_1 to estimate greater than one as this would imply that producers were paying labor more than its marginal product, causing profits to fall.

Equating (4d) and (4e), taking the natural log of both sides, and solving for E, we obtain:

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[4]Because elsewhere in the model the symbol W represents the wage **bill** and not the wage **rate**, the real wage rate will be expressed as W/EP throughout the rest of the paper.

$$(4f) \quad \ln E_t = \ln \beta_2 + \ln Y_t - \phi_1 \ln \frac{W_t}{E_t} + \phi_1 \ln P_t$$

In this formulation we expect a unit coefficient on Y and the size of ϕ_1 tells us how close we are to the real wage equating marginal product condition. This form of the employment equation takes account only of the negative sloping portion of the inverted "v" where employment is a negative function of the real wage. We must also attempt to incorporate a variable in the employment equation that will account for the positively sloped portion.

We do this by noting that if the economy is at a position to the left of (W/P)*, aggregate demand determines the level of output and the level of employment. At this level of output the real wage is less than productivity, or $W/EP - MPP_e < 0$. To account for this in our function we add productivity to the employment equation as a corrective factor. In the region on the left leg of the inverted "v", employment and the real wage increase together while productivity decreases as the economy moves toward equilibrium in the commodity market. In the region on the right leg of the inverted "v", employment and the real wage move in opposite directions while employment moves with productivity. We would hope to get a negative empirical relationship between employment and productivity if we are in fact picking up some of the influence of the positive relationship between the

real wage and employment during periods where aggregate demand controls the productive decisions.

Adding the productivity variable does something else for the employment equation. It allows it to shift over time in response to shifts in the production function. The employment function takes on the desired long run characteristics by allowing for shifts in the function over time. In this type of a formulation we are implicitly assuming disembodied or neutral technical change. A negative sign on productivity in this context would suggest capital-labor substitution.

Since we can find no a priori reason prohibiting the inclusion of productivity as additive in the logs, we modify (4f) as follows:

$$(4g) \quad \ln E_t = \pi_1 \ln \beta_2 + \pi_2 \ln Y_t - \pi_3 \ln \frac{W_t}{E_t} + \pi_4 \ln P_t$$

$$- \pi_5 \ln A_t$$

The reader will note the change in coefficients between Equation (4f) and (4g). Because of the interdependence of wages, output, and productivity, we could not expect the coefficients to remain the same after the inclusion of productivity in the equation. In the empirical estimation of this equation we would no longer expect unitary coefficients on $\ln \beta_2$, and $\ln Y$, nor would we expect identical

coefficients on wages and prices.

The estimated equation is as follows:

$$\ln E_t = -3.713 + .833 \ln Y_t - .424 \ln W_t/E_t + .743 \ln P_t$$
$$(11.32) \quad (18.93) \qquad (6.51) \qquad\qquad (9.48)$$

$$- .478 \ln A_t \qquad \bar{R}^2 = .999$$
$$(13.78)$$
$$DW = 1.56 \quad \rho = .54$$

The equation was estimated using generalized least squares. The equation coefficients come with the expected signs. There is a strong relationship between output and employment as would be expected. The coefficients on W/E and P are different from each other suggesting that productivity has a different relationship with wages and with prices. The negative sign on A is difficult to interpret because of the dual function of A in the equation. The negative sign supports a capital-labor substitution effect, as well as the argument of productivity decrease and employment increase as the economy moves from a position of insufficient aggregate demand toward equilibrium.

3. The wage function

Since A. W. Phillips (89) established an empirical relationship between wages and unemployment for the U. K., attempts to improve and broaden his simple specification of the wage function have been prevalent in the literature.

Beginning with the simple Phillips relationship, further study has produced considerable conflicting evidence. In an attempt to derive a Phillips Curve for the United States. Samuelson and Solow (94) published a scatter diagram for aggregate data indicating no relationship between wages and unemployment. In a later study, Rees and Hamilton (92) found an unstable Phillips curve, and offered the possibility of the existence of a family of Phillips curves.

In an attempt to discover a more stable relationship between wages and unemployment, Lipsey and Steuer (74) added profits to the function as an explanatory variable. Using annual data they found the addition of the profit variable did improve the explanatory ability of the function, and also found that profits and unemployment have about equal explanatory power.

Following this study, Perry (86) published an extensive study relating wages, lagged prices, unemployment, and various profit variables and was able to explain an impressive amount of variation in changes in wages in the manufacturing sector. His reported R^2 is .87. The inclusion of lagged prices in a model containing profits and unemployment to explain wage changes, and the use of quarterly data to estimate the model, improve the explanation of wage changes in U.S. manufacturing.

What these and similar studies have in common is that they are mainly empirical investigations into wage determination without theoretical underpinnings. In fact Perry (86)

says: "No attempt is made here to justify formally the hypothesis that these variables are important."[5]

Edwin Kuh (67) attacks the Perry profit hypothesis in a theoretical and empirical work that offers productivity as the key determinant of changes in the wage rate, arguing that profits, in fact, are merely a proxy for productivity. The amount of variation in wage changes that Kuh is able to explain using his formulation is only about half that ex- plained by Perry.

Christian (22), in formulating his wage equation to determine the effectiveness of the wage-price guideposts, incorporates a productivity variable along with prices and unemployment. In the manufacturing sector he is able to generate an $\bar{R}^2$ of .93, which exceeds that of Perry (86). In this paper, Christian compares the average forecast error in a model using a profit variant with the average forecast error using the productivity variant. He reports a forecast error in the profit variant models one and one- half times as large as the errors in the productivity variant formulation.

In our development of the wage equation, wages are seen

[5]G. L. Perry, "The Determinants of Wage Rate Changes and the Inflation-Unemployment Trade-off for the United States", Review of Economic Studies, 31, (Oct. 1964), p. 287.

as a factor price. Because they are a price, wages are determined by the interplay of the forces of supply and demand. First, a demand for labor function is specified, then a supply of labor function is derived. Then the two are set equal and wages are isolated, giving us the wage function used in this model.

The demand for labor formulation follows the ideas presented by Kuh (67), in that productivity is the major determinant in the demand for labor. Following the same derivation as the employment equation, the Cobb-Douglas production function is again postulated:

$$\text{(5a)} \quad Y = AK^{\beta_1} E^{\beta_2}$$

the marginal product of labor is derived from this function and equated to the real wage:

$$\text{(5b)} \quad \beta_2 \, Y/E = (W/EP)^{\phi_1}$$

natural logs are taken and the employment variable is again algebraically isolated:

$$\text{(5c)} \quad \ln E^D = \ln \beta_2 + \ln Y - \phi_1 \ln \frac{W}{E} + \phi_1 \ln P$$

and treated as the employer's demand for labor.

A supply of labor function is specified as follows:

$$\text{(5d)} \quad E^S = (W/EP)^{\gamma_1} (N)^{\gamma_2} (U)^{\gamma_3}$$

W/EP is again the real wage, N is the size of the population between the ages of 14 and 64, and U is the level of unemployment. It is hypothesized that γ_1 will be positive, as one would expect a higher real wage to attract a larger supply of labor. γ_2 is also postulated as positive. As the size of the potential labor force grows, the supply of labor should also increase. However, the postulated sign of γ_3 is unclear. As U increases this may have the effect of either increasing or decreasing the supply of labor, depending on whether the "additional worker" or "discouraged worker" effect is dominant. The "discouraged worker" effect, as presented by Strand and Dernberg (109), derives from the case in which as unemployment rises and output falls, some workers are discouraged by not being able to find work and drop out of the labor force. As unemployment rises the supply of labor falls. Conversely, as unemployment falls, the supply of labor will rise as these workers are now encouraged to re-join the labor market. The "additional worker" effect is defined as the case in which as unemployment increases, secondary workers are drawn into the labor force by economic need. Thus, an increase in unemployment fosters an increase in the labor force. There remains the possibility that no relationship exists between the level of unemployment and the size of the labor force if the two effects cancel each other

out. The current literature has no definitive evidence
for either conclusion. The current state of opinion is that
there is a differential effect present at different times
depending on the intensity and length of recession or boom.

The unemployment variable may also have the effect, in
addition to shifting the supply of labor curve, of changing
its elasticity. In periods of low unemployment the curve
may become very inelastic. During periods of low unemploy-
ment, the result of the increase in the demand for labor
is an increase in the real wage. Conversely, when unemploy-
ment is high, the supply of labor curve becomes increasingly
elastic, and any increase in the demand for labor can be met
without large increases in the real wage. In terms of an
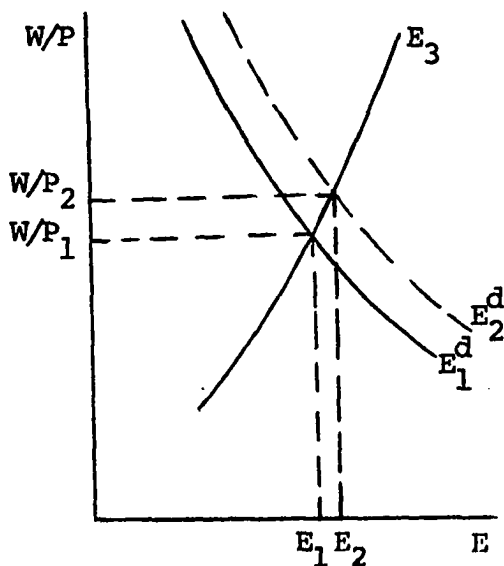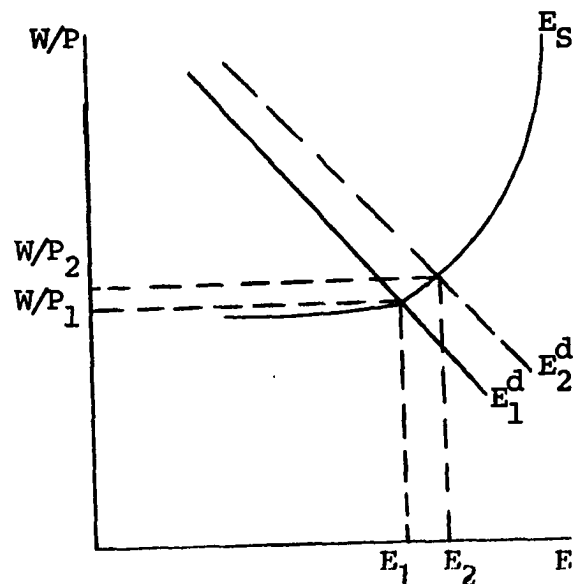illustration:

Figure 1.   Low U

Figure 2.   High U

In Figure 1, low unemployment has caused the supply of labor function to become very inelastic. Any increase in the demand for labor, for example from $E_1{}^d$, to $E_2{}^d$, will cause a large rise in the real wage to increase employment. In Figure 2, higher unemployment causes the supply of labor curve to become more elastic. In response to an increase in demand from $E_1{}^d$ to $E_2{}^d$, more labor can be employed with little change in the real wage. It is in this manner that unemployment can change the elasticity of the supply of labor function.

Therefore, using the supply of labor function specified in Equation (5d) and taking the natural log of this equation we obtain:

(5e) $\quad \ln E^S = \gamma_1 \ln \frac{W}{E} - \gamma_1 \ln P + \gamma_2 \ln N + \gamma_3 \ln U$

Setting the demand for labor (5c) equal to the supply of labor (5e), and solving for ln W/E, the wage equation is obtained:

(5f) $\quad \ln \frac{W}{E} = \frac{1}{\gamma_1 + \phi_1} \ln \beta_2 + \frac{1}{\gamma_1 + \phi_1} \ln \gamma + \ln P - \frac{\gamma_2}{\gamma_1 + \phi_1} \ln N$

$$- \frac{\gamma_3}{\gamma_1 + \phi_1} \ln U$$

Because this is a simultaneous system of equations, it would be unrealistic to take the unemployment level as an exogenous variable. We therefore endogenize unemployment in the following manner. First we specify the following

identity:

(5g) U = L-E

where U, the level of unemployment, is defined as the dif-
ference between the labor force L, and the level of employ-
ment, E. Then dividing both sides of the equation by L we
obtain:

(5h) U/L = 1 - E/L

Now define another identity:

(5i) (1-E/L) = (E/L)$^{\sigma}$

where σ is that number that makes this an identity. For
example if E/L = .5 then σ would equal 1.00. As E/L in-
creases, σ remains positive and becomes greater than one.
Now, substituting the right hand side of (5i) into (5h),
the following is obtained:

(5j) U/L = (E/L)$^{\sigma}$

Taking the natural log of both sides and solving for ln U:

(5k) ln U = σ ln E + (1-σ) ln L

We now specify that:

(5l) L = N$^{\upsilon}$

where ν is the elasticity of the labor force with respect to population. Again taking the natural log of this equation:

(5m) $\ln L = \nu \ln N$

Making the proper substitution of $\ln L$ into (5k), we obtain our expression for unemployment:

(5n) $\ln U = \sigma \ln E + (1-\sigma)\nu \ln N$

Substituting this equation for unemployment into the wage equation, we obtain:

(5o) $\ln \dfrac{W}{E} = \dfrac{1}{\gamma_1+\phi_1} \ln \beta_2 + \dfrac{1}{\gamma_1+\phi_1} \ln Y$

$$+ \ln P - \frac{\{\gamma_2+\gamma_3\nu(1-\sigma)\}}{\gamma_1+\phi_1} \ln N - \frac{\gamma_3\sigma}{\gamma_1+\phi_1} \ln E$$

Because many policy discussions focus on the wage income rather than an aggregate wage rate, we express this equation in terms of the wage bill. Noting that: $\ln W/E = \ln W - \ln E$, we add $\ln E$ to both sides of (5o), obtaining the wage equation included in the model:

(5p) $\ln W_t = \dfrac{1}{\gamma_1+\phi_1} \ln \beta_2 + \dfrac{1}{\gamma_1+\phi_1} \ln Y_t$

$$+ \ln P_t - \frac{\{\gamma_2-\gamma_3\nu(1-\sigma)\}}{\gamma_1+\phi_1} \ln N_t + \{1 - \frac{\gamma_3\sigma}{\gamma_1+\phi_1}\}\ln E_t$$

The estimated equation is as follows:

$$\ln W_t = 2.871 + .714 \ln Y_t + 1.202 \ln P_t - .0032 \ln N_t$$
$$ (2.634)\ (9.476) (18.082) (.066)$$

$$+ .609 \ln E_t \qquad \bar{R}^2 = .999$$
$$ (5.238)$$
$$ DW = 1.821 \quad \rho = .69$$

Because of the low Durbin-Watson using ordinary least squares, the equation was estimated using generalized least squares. The signs of the variables in the estimated equation are as predicted for output, prices, and population. The positive sign on employment indicates that the sign of γ_3, the elasticity of the supply of labor with respect to unemployment must be positive, since the denominator, $(\gamma_1 + \phi_1)$, is known to be positive from the positive coefficient on ln Y, and since σ must also be positive and greater than 1. In order for the expression $[1 - \dfrac{\gamma_3 \sigma}{\gamma_1 + \phi_1}]$ to be positive and less than 1, γ_3 must be positive. This evidence supports the "additional worker" hypothesis. To generate further confirmation on the sign of γ_3, the equation was reestimated using W/E as the dependent variable. In this formulation the coefficient on employment becomes $- \dfrac{\gamma_3 \sigma}{\gamma_1 + \phi_1}$ ln E. The sign of the estimated coefficient of ln E in this form of the equation is negative further supporting the positive sign of γ_3.

The coefficient on prices is greater than unity, the theoretical size of this coefficient. We are, perhaps,

picking up the "over-reaction" by labor to inflation in the
latter part of the 1960's. Given an inflationary period and
the expiration of labor contracts, labor attempts to make up
for past losses as well as projected future losses in pur-
chasing power. Therefore, the coefficient on prices would
be greater than unity. To support this interpretation, we
estimated the wage equation for the first 36 quarters of the
sample period using generalized least squares. This esti-
mation of the wage equation covers the period from 1952 to
1960, which is a period of relative price stability. For
this period the coefficient on prices falls to .998. The
hypothesis of the over-reaction of wages to price increases
during inflationary periods is empirically supported. The
entire estimated equation for wages from 1952-1960 is pre-
sented in Appendix B.

It is impossible to disentangle the coefficient on Y.
It is readily determined that the value of $(\phi_1 + \gamma_1)$ is 1.4.
Given that ϕ_1 is the indicator of the extent to which labor
receives its marginal product, we know that its value must
be positive and, in all likelihood, cannot, exceed unity.
Thus, if ϕ_1 is equal to, or very close to, unity, γ_1, the
elasticity of the supply of labor with respect to the real
wage would be equal to, or very close to .4. Without
independent information, we can only conclude that this
is a reasonable estimate.

We attribute the non-significance of the coefficient of population to the small variance in the series compared with changes in the wage bill.

Expectations by labor about the future rate of inflation plays a pivotal role in the wage-price nexus. Thus it would be most desirable to incorporate a measure of expectations into the model. An initial attempt to incorporate expectations into the wage equation is presented and discussed in Appendix C.

E. The Complete Model

Before proceeding to the derivation of the reduced form, it will be useful to summarize the complete theoretical model and the estimates of its structural equations:

Theoretical Model:

(1) $\ln P_t = \ln M_t + \ln V_t - \ln Y_t$

(2) $\ln V_t = \alpha_{21} \ln \beta_o + \alpha_{22} \ln Y_t + \alpha_{23} \ln P_t - \alpha_{24} \ln P_{t-1}$

(3) $\ln [Y/E]_t = \ln A_t + \beta_1 \ln [K/E]_t$

(4) $\ln E_t = \alpha_{41} \ln \beta_2 - \alpha_{42} \ln A_t + \alpha_{43} \ln Y_t$

$\qquad - \alpha_{44} \ln [W/E]_t + \alpha_{45} \ln P_t$

(5) $\ln W_t = \alpha_{51} \ln \beta_2 + \alpha_{52} \ln Y_t + \alpha_{53} \ln P_t$

$\qquad + \alpha_{54} \ln N_t + \alpha_{55} \ln E_t$

(6) $\ln Y_t = \ln [Y/E]_t + \ln E_t$

(7) $\ln [W/E]_t = \ln W_t - \ln E_t$

(8) $\ln [K/E]_t = \ln K_t - \ln E_t$

Structural Estimates:

(2) $\ln V_t = -1.9385 + .5139 \ln Y_t - 3.106 \ln P_t$

$\qquad + 3.817 \ln P_{t-1}$

(3) $\ln Y/E_t = 5.6617 \ln A_t + .832 \ln [K/E]_t$

(4) $\ln E_t = -3.713 + .833 \ln Y_t - .424 \ln [W/E]_t$

$\qquad + .743 \ln P_t - .478 \ln A_t$

(5) $\ln W_t = 2.878 + .714 \ln Y_t + 1.202 \ln P_t - .0032 \ln N_t$

$\qquad + .609 \ln E_t$

The output equation, the employment equation aht eht wage equation are all estimated using generalized least squares. The general form of an equation estimated by GLS is:

$$Y_t = \alpha_0 + \alpha_i X_{it} + \rho u_{t-1} + e_t$$

The structural equations to be used in the computation of the reduced form coefficients must be of this form. We, therefore, solve for u_{t-1} from the original equation:

$$U_{t-1} = Y_{t-1} - \alpha_0 - \alpha_i X_{i_{t-1}}$$

and substitute this into the above expression:

$$Y_t = \alpha_0(1-\rho) + \alpha_i X_{i_t} + \rho Y_{t-1} - \rho \alpha_i X_{i_{t-1}} + e_t$$

Each structural equation estimated by GLS now contains not only the current values of the relevant variables, but ρ times the lagged values of the independent and dependent variables as well.

Also before proceeding to the reduced form of the model and the policy implications that can be generated, several comments about the model seem in order. This is a very small, very aggregative, and very interdependent model. It lacks a financial sector, and endogenous capital stock, an indicator for government fiscal policy, and in independent foreign sector. With this in mind, the following discussion of the reduced form, its coefficients, and the policy implications derived from it should be understood as an indication of the true relationship existing among the variables discussed.

III. THE DERIVED REDUCED FORM OF THE MODEL, ITS RELIABILITY,

AND THE POLICY IMPLICATIONS

A. The Derived Reduced Form of
the Model

Once we have estimated each of the structural equations,
getting the individual effect of each of the independent
variables on each of the dependent variables, the next step
is to take the model into its reduced form. The inter-
dependence of the entire system is then taken into account
and the effect of each exogenous variable on each endogenous
variable is isolated.

In matrix notation, we begin with:

$$\Gamma Y = -\beta X$$

where Y is the vector of jointly determined variables, X the
vector of exogenous variables, Γ the matrix of coefficients
of the jointly determined variables, and β the matrix of
coefficients of the exogenous variables.

We then calculate Γ^{-1}, and premultiply both sides of
the matrix equation by Γ^{-1}, giving us the derived reduced
form:

$$Y = \pi X$$

Table 1 presents the π matrix of reduced form coeffi-
cients. The endogenous and exogenous variables are so labeled

Table 1. Matrix of reduced form coefficients

Exogenous variables	(Endogenous Variables)							
	P	V	Y/E	E	W	Y	W/E	K/E
Con	-.4138	-.9057	2.4363	-2.928	-1.742	-.4919	1.1864	2.9282
M	.24204	-.7452	-.0631	.0758	.3462	.0127	.2704	-.7058
A	-.0745	.5546	.8231	-.1939	.2416	.6291	.4355	.1939
N	-.00004	.0003	-.0015	.0018	-.0019	.0003	-.0037	-.0018
K	-.1096	.8161	.3679	.5578	.8689	.9257	.3111	.4422
P-1	.9251	.9626	-.1857	.2232	.4453	.0375	.2221	-.2232
W-1	.0078	-.0578	.3246	-.3902	.4149	-.0656	.8051	.3902
Y-1	.0064	-.0476	.2674	-.3213	-.7192	-.0540	-.3979	.3213
A-1	.0742	-.5526	-.5586	-.0683	-.3999	-.6269	-.3317	.0683
E-1	-.0191	.1419	-.7969	.9579	.2551	.1609	-.7027	-.9579
N-1	.00002	-.0002	.0010	-.0012	.0013	-.0002	.0026	.0012
W/E-1	-.0061	.0452	-.2541	.3054	.2153	.0513	-.0901	-.3054
Y/E-1	-.1225	.9122	.4113	.6235	.9712	1.0347	.3478	-.6235
K/E-1	.1019	-.7590	-.3422	-.5187	-.8081	-.8609	-.2894	.5187

in the table. Each coefficient in the π matrix represents, ceteris paribus, the change in an endogenous variable given a unit change in an exogenous variable. For example, if the money supply increases by 1%, prices would increase by .24205%, etc.

In this model, our basic idea was to isolate the money supply as a policy variable in order to determine the effect of changes in the money supply on prices, wages, employment, and output. Looking at the reduced form coefficients indicates first that a unit increase in the money supply will increase prices and the wage rate by an almost equal amount (.27 for the wage rate vs .24 for prices). Money supply changes have their strongest effect on current velocity (-.75). The strong negative effect suggests the existence of a lag in the effect of changes in the money supply as reported by Friedman and others. What we cannot discern from the model in its current specification, is the effect on the real variables of the system that will take place as actual money balances are brought into equilibrium with the desired level of money balances. Lags aside, if these esti- mates are at all indicative of the true relationships, the small impact of changes in the money supply on output (.012) and employment (.08) over the period studied is rather striking. A possible explanation of these small coefficients is that the rapid rate of growth in the money supply during

the latter part of the 1960's coincided with a period of full

employment, and full production. The coefficients during

this period may well dominate the full period relationship,

giving the result that changes in the money supply really

affect only prices and wages and do not affect output

and employment significantly. To check this possibility the

model should be reestimated over a period of relative price

stability, and the coefficients compared. Alternatively, the

estimates provide support for the proposition that money is

neutral. This is not necessarily surprising. These esti-

mates represent an 18 year average of the quarterly effect

of changes in the money supply on output and employment,

and we may be picking up the long run neutrality of money.

The reduced form further reveals that a unit increase in

productivity has a small (-.075) negative effect on prices

and a moderately negative (-.19) effect on employment, while

increasing the wage bill (.24), output (.63), and the wage

rate (.45). The signs and magnitudes of these relationships

are as expected. The small effect on prices is disappointing,

but may be explained by the fact that increases in productiv-

ity more often go into increases in wages rather than de-

creases in prices. The coefficient on employment (-.19)

suggests capital-labor substitution, and/or that the relation-

ship between increases in productivity and employment during

the upward movement on the cycle are overpowered by the in-

creases in productivity gained as marginal workers using older capital equipment are laid off as a recession begins. The strong effect (.61) on output indicates the importance of productivity gains for noninflationary expansion.

Changes in population show little effect on any of the endogenous variables.

Capital formation, as expected, is clearly a key variable. A unit increase in the capital stock has a moderate effect on decreasing prices (-.11), increasing employment (.56), and increasing the wage rate (.31), and a strong effect on increasing the wage bill (.87), output (.92), and velocity (.81).

Turning now to the lagged coefficients, the interplay of lagged prices and lagged wages is interesting. Initially it appears that price increases last quarter will increase employment (.22), or inflation is beneficial to employment. This view ignores the relationship between wages and prices. Although increasing prices last quarter do have a positive effect on employment, increasing wages last quarter will cause a decline (-.39) in current employment. From the structural equation for the determination of the wage bill, we note that current wage increases are 1.2 times as large as current price increases. This suggests that a 1% increase in prices this quarter, and a 1.2% increase in wages this quarter should lead to a net decline in employment next quarter, if we con-

sider only the wage-price effects on employment, of -.24%.
Isolating the wage-price effect on future employment indi-
cates that inflation really does not increase employment, but
has the opposite effect. As would be expected given the above
discussion of the employment effect, the net effect of in-
creasing wages and prices on the next quarter's real output is
also negative, but the net effect is small (-.08%). Combining
the wage and price lags with the money supply, it is noted
that real output is unresponsive to changes in the monetary
variables of the system.

The effect of lagged prices on the monetary variables of
the system is very strong, affecting current prices with a
coefficient of .92, velocity with a coefficient of .96, and
the current wage bill with a coefficient of .45. The strong
response of velocity to lagged price increases suggests that
although the monetary authorities control the money supply in
an attempt to control inflation, increases in velocity during
an inflationary spiral can lead to still further inflationary
pressures on the economy. The rate of growth of the money
supply may have to be slowed even more than originally thought
to halt inflation.

Considering the coefficients on lagged wages, we find
little evidence to verify a wage push element in the in-
flationary spiral, as the effect of lagged wages on current
prices is very small (.008).

The lagged output coefficients suggest that the higher output last quarter, the lower (-.32) employment in the current period. The employment effect could explain the negative coefficient between lagged output and the wage bill (-.72) and the wage rate (-.40).

The coefficients on lagged productivity are very puzzling. An increase in productivity this quarter leads to price decreases (-.07) in the current period, but price increases (.07) in the following quarter. As would be expected, lagged productivity leads to further decreases in employment (-.07). It also leads to decreases in the wage bill (-.40), output (-.63) and the wage rate (-.33). Again we may be picking up the employment effect on productivity increases that could account for these rather odd results.

B. The Reliability of the Model

In order to determine the reliability of the model, the data were tracked over the sample period using the derived reduced form equations. These estimates were compared with the actual values of the data used in the estimation of the model. The methodology of this test is as follows. We begin with the reduced form of the model where each endogenous variable is specified as a function of all the exogenous variables multiplied by the proper reduced form

coefficient. In matrix notation this becomes: $Y = \pi X$.

Using these reduced form equations, we generate 72 quarterly predictions for each of the endogenous variables in the model, $\hat{Y}_{ij}$, where "i" subscripts the year and "j" subscripts the quarter of the forecast variable. To generate the forecasts, $\hat{Y}_{ij}$, the values of the exogenous variables that were used to estimate the model are read in, and multiplied by the appropriate reduced form coefficient. For example:

$$\hat{P}_{52:I} = con + \pi_{1,2}M_{52:I} + \pi_{1,3}A_{52:I} + \pi_{1,4}N_{52:1} + \pi_{1,5}K_{52:I}$$

$$+ \pi_{1,6}P_{51:IV} + \pi_{1,7}W_{51:IV} + \pi_{1,8}Y_{51:IV} + \pi_{1,9}A_{51:IV}$$

$$+ \pi_{1,10}E_{51:IV} + \pi_{1,11}N_{51:IV} + \pi_{1,12}W/E_{51:IV}$$

$$+ \pi_{1,k3}V/E_{51:IV} + \pi_{1,14}K/E_{51:IV} .$$

or

$$\hat{P}_{52:I} = -.41376 + .2420\,M_{52:I} - .0745\,A_{52:I} - .00004\,N_{52:I}$$

$$- .1096\,K_{52:I} + .9251\,P_{51:IV} + .0078\,W_{51:IV}$$

$$+ .0064\,Y_{51:IV} + .0742\,A_{51:IV} - .0191\,E_{51:IV}$$

$$+ .00002\,N_{51:IV} - .0061\,W/E_{51:IV} - .1225\,Y/E_{51:IV}$$

$$+ .1020\,K/E_{51:IV}$$

A vector of $\hat{Y}_{ij}$ is computed for each of the endogenous variables. The next step is to run a correlation on Y and $\hat{Y}$, using the formulation:

$$Y = a + b\hat{Y}$$

to compare the forecast values with the actual values.
Finally, we test the hypothesis that a = 0 and b = 1 from
the correlation analysis. If a does not significantly differ
from 0, and b does not significantly differ from 1, the
model tracks the data very well over the sample period.

The results of this test are presented in Table 2.
At the .01 level of significance we are able to accept the
null hypothesis of a = 0 or b = 1 for 10 of the 16 correla-
tions. This evidence suggests the model tracks the data
reasonably well over the sample period.

C. The Policy Implications from
the Reduced Form

In order to derive the full policy implications of the
model, a dynamic simulation of the model needs to be computed
and the long run or dynamic policy multipliers derived.
From this type of an analysis the optimum rate of growth of
the money supply could be ascertained, given the policy ob-
jectives.

This extension of the analysis is beyond the scope of
the current work. We can, however, arrive at a general idea
of the implications for employment and real output that a
rate of growth of the money supply consistent with price
stability would imply. This type of analysis is not

Table 2. OLS reduced form "forecast" analysis

Dependent variable	a	"t" value H: a≠0	b	"t" value H: b-1≠0	$\bar{R}^2$	D.W.	S.E.E.	C.V.
P	.00032 (.00069)[a]	.464	1.0015 (.0061)	.246	.997	.418	.0056	.168
V	.0200 (.008)	2.5	.9852 (.0062)	2.258	.997	.785	.0092	.007
Y/E	.1143 (.0574)	1.99	.9871 (.0064)	2.08	.997	1.546	.0064	.001
E	-.0619 (.0209)	2.96	.9761 (.0077)	3.10	.996	1.134	.0052	-.002
W	.0647 (.0202	3.2	.9895 (.0035)	3.00	.999	.7002	.0095	.002
Y	.1010 (.0321)	3.14	.9841 (.0051)	3.11	.998	.942	.0086	.001
W/E	.0563 (.0306)	1.83	.9935 (.0036)	1.81	.999	1.409	.0073	.001
K/E	-.1315 (.0579)	2.27	1.0134 (.0060)	2.23	.997	1.102	.0054	.001

[a]Standard errors in parentheses.

definitive, but is merely suggestive of the kind of policy

implications this model is capable of generating.

We begin this analysis by computing a trade-off matrix

which is presented in Table 3.[7] The coefficients in this

table represent the compensating change in the money supply

required to hold an endogenous variable constant, given a 1%

change in an exogenous variable in one time period. For

example, if there is a 1% increase in productivity, the money

supply must increase by .3076% in order to hold prices

constant, by .7443% to hold velocity constant, by 12.9188%

to keep the output-employment ratio constant, etc. The method

used to compute this table is presented in Appendix D.

Among the more interesting coefficients in the table, are the

changes in the money supply necessary to compensate for

changes in certain exogenous variables in order to keep real

output constant. For example, if the capital stock declines

by 1%, the table indicates that it would take a .72718% in-

crease in the money supply to maintain real output at its

original level in one time period. This coefficient reflects

the large response of real output to changes in the capital

stock (.93), and the minimal response of real output to changes

in the money supply (.013). This minimal response of output

to money supply changes supports an argument for the

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[7]This type of a trade-off approach was employed by DeWald
and Johnson (29) to provide an analysis of Federal Reserve
policy making.

Table 3. Tradeoff matrix (this table represents the change in the money supply needed to hold an endogenous variable constant given a unit change in A, K, N, or P_{t-1}

	P	V	Y/E	W	E	Y	K/E	W/E
A	.3076	.7443	12.9188	-.6977	2.5584	-49.421	2.5584	-1.6106
K	.4528	1.095	5.834	-2.5098	-7.3583	-72.718	5.343	-1.1506
N	.0002	.0004	-.0239	.0055	-.0239	-.0236	-.0239	.0138
P_{t-1}	-3.822	1.2917	-2.9445	-1.2864	-2.9446	-2.946	-2.9446	-.8275
W-1	-.0321	-.0775	5.1473	-1.1984	5.1478	5.1493	5.1478	-2.9775
Y-1	-.0266	-.0639	4.2389	2.0775	4.2393	4.241	4.2393	1.4715
A-1	-.3066	-.7416	-8.8564	1.1553	.9008	49.242	.9008	1.2266
E-1	.0793	.1904	-7.6356	-.7369	-7.6365	-12.6410	-7.6365	2.5989
N-1	-.0001	-.0002	.0165	-.0038	.0164	.016	.0164	-.0095
W/E-1	.0251	.0607	-4.0284	-.6219	-4.6286	-4.030	-4.0028	.3331
Y/E-1	.5062	1.2241	6.521	-2.8055	-8.2251	-81.284	-8.2251	-1.2862
K/E-1	-.4211	-1.0185	-5.4256	2.3342	6.8433	-67.628	6.8433	1.0701

neutrality of money, or an argument for significant lags in the effect of monetary policy.

Now, combining Table 3 and the reduced form coefficients in Table 1, we find that if we know the quarterly growth rates of productivity and the capital stock, inflation last quarter, (because population effects are so minute they will be omitted from the discussion) the rate of growth in wages last quarter, output last quarter, productivity last quarter, employment last quarter, the wage rate last quarter, output per man last quarter and capital per man last quarter, we can determine the rate of growth in the money supply required to maintain price stability. If we assume as a starting point a period of price stability, then the rate of growth of the money supply needed to maintain price stability can be determined from the following relationship:

(a) $\Delta M = .3078 \ \Delta A + .453 \ \Delta K - .032 \ \Delta W_{t-1} - .027 \ \Delta Y_{t-1}$

$- .307 \ \Delta A_{t-1} + .079 \ \Delta E_{t-1} + .025 \ \Delta W/E_{t-1}$

$+ .506 \ \Delta Y/E_{t-1} - .421 \ \Delta K/E_{t-1}$

Once we have determined the change in the money supply in this fashion, we can then turn to the reduced form coefficients in Table 1 to ascertain the rate of growth in employment and output implied by the growth rates in the relevant variables using the following relationships:

(b) $\Delta E = .0759 \Delta M - .1936 \Delta A + .558 \Delta K - .390 \Delta W_{t-1}$

$- .321 \Delta Y_{t-1} - .068 \Delta A_{t-1} + .958 \Delta E_{t-1}$

$+ 305 \Delta W/E_{t-1} + .6230 \Delta Y/E_{t-1} - .519 \Delta K/E_{t-1}$

(c) $\Delta Y = .0127 \Delta M + .6293 \Delta A + .926 \Delta K - .066 \Delta W_{t-1}$

$- .054 \Delta Y_{t-1} - .627 \Delta A_{t-1} + .161 \Delta E_{t-1} + .051 \Delta W/E_{t-1}$

$+ 1.035 \Delta Y/E_{t-1} - .861 \Delta K/E_{t-1}$

To illustrate, assume that the exogenous and lagged endogenous variables take the following values: A grows at .75% per quarter, K at .75% per quarter, Y_{y-1} at 1% per quarter, A_{t-1} at .5% per quarter, E_{t-1} at .5% per quarter, W_{t-1} at 1% per quarter, W/E_{t-1} at .8% per quarter, Y/E_{t-1} at .5% per quarter, and K/E_{t-1} at .4% per quarter. We then compute from relationship (a) that the rate of growth of the money supply consistent with price stability is .53% per quarter, or an annual rate of 2.1% per year. We note that this growth rate is close to the 2% annual growth rate proposed by Milton Friedman (48), and we, therefore, feel that the model tends to generate reasonable values for policy purposes.

Using the rate of growth of the money supply calculated in (a) to be consistent with price stability, we turn to

relationships (b) and (c) to determine the rates of growth of employment and real output that would also occur. We find that employment will grow at about .4% per quarter or at an annual rate of 1.6%, and will slightly exceed the average annual rate of growth of the labor force of 1.56%. Real output will grow at about 1.1% per quarter or 4.4% per year.

If a faster rate of growth in real output or employment is sought, the reduced form coefficients point to the capital stock as the key variable. Capital stock increases should also lead to productivity increases which would further increase output, but lead to declines in employment. The net effect on employment will still be positive unless productivity increases by 3% for every 1% increase in the capital stock. The data we have on the capital stock and productivity do not suggest such a relationship.

In concluding this section, we note that from the reasonable coefficients obtained from this feasibility exercise, that the model is well worth continued effort, and that simulations and dynamic multipliers would be worth-while calculating for their more realistic policy implications.

V. SUMMARY, CONCLUSIONS, AND SUGGESTIONS

FOR FUTURE RESEARCH

In the present paper, we have specified and estimated a quarterly macroeconometric model in a way that would allow the effect of changes in the money supply to directly influence prices, employment, wages, and output, without forcing the money mechanism to work through an interest rate.

The formulated model is divided into two subsectors, the demand subsector and the supply subsector. The demand sector consists of the exogenous money stock and the velocity equation. The supply sector consists of the aggregate production function, an employment function and a wage function. The model is closed by three definitional identities. Each of the structural equations was estimated by either ordinary least squares or generalized least squares over the period 1952-1969 using quarterly observations.

The major conclusions of the model were drawn from the derived reduced form of the model. Changes in the money supply over the 18 year estimation period were observed to have their most significant effect on velocity. This finding strongly suggests a substantial lag in the effect of monetary changes on the economic variables of the system. Changes in the money supply had a quantitatively similar

effect on wage rates and prices, suggesting a weak response of the real wage to monetary changes. Finally, changes in the money supply had little impact on output and employment, supporting the proposition of long run money neutrality.

In the area of policy implications it was noted that to extract the real policy implications from the model simulations should be done, and the long run as dynamic multipliers computed. A feasibility example was presented to test the quality of the model in generating reasonable policy recommendations. From this simple example employing a trade-off matrix and the reduced form coefficients we computed that the rate of growth of the money supply consistent with price stability was 2.1% annually. This estimate compares favorably with that rate of growth presented by Milton Friedman, and therefore seems reasonable. We also computed that the growth in employment would slightly exceed the growth rate of the labor force, and the growth in output would be about 4% annually. Both are reasonable values also. Further, from the reduced form it was easily seen that the rate capital formation is the key variable in increasing growth rates for both real output and employment while maintaining price stability.

In terms of future theoretical development, much can be done to improve the present model. A framework to incorporate monetary lags would be desirable as would a full financial

sector. The capital stock should be endogenized and expectations should be added.

In terms of future empirical work, the model should be extended into the future to test its predictive accuracy from 1970 to 1972. It would also be most interesting to reestimated the model for periods characterized by un-utilized resources to determine if in these periods the neutrality of money is maintained. Also in the interest of determining monetary neutrality, a short term cyclical analysis might also be attempted. It has not escaped our notice, that a refined version of this model might be used to separate the income and price effects of monetary changes.

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VII. APPENDIX A

The data to estimate this model are drawn from several sources. The primary source of data was Data Resources Incorporated (DRI). The price level, P, is the GNP deflator and its retrieval code from DRI is PGNP. Velocity is computed as the ratio of Gross National Product to the money supply. The retrieval code for Gross National Product is GNP. Money supply is defined as currency plus demand deposits. Its retrieval code is MONEY1. This is a monthly series on DRI, and was made quarterly using averages. Real output, Y, is measured as Gross National Product in constant dollars. Its retrieval code is GNP58. The price level lagged one quarter, P_{t-1}, is the GNP deflator lagged one quarter. The wage bill, W, is measured as total compensation to employees. Its retrieval code is WSS. Employment, E, is the number of men and women employed. Its retrieval code is EHH.

A second source of data is that used to estimate the FRB-MIT-Penn quarterly macroeconomic model of the United States economy. Our series for capital, K, is drawn from these data. To get an aggregate measure for capital, we summed the series for capital in producer's structures, 18; capital in producer durables, 30; capital owned by state and local governments, 102; the capital measure of single family

dwellings, #294; and the capital measure of multiple family dwellings, #296. The measure for the level of technology is proxied by an output per man-hour variable. The data to compute this proxy also comes from the FRB-MIT-Penn model. The series for real output, #3, was first adjusted from an annual rate to a quarterly rate. Then this adjusted series for output was divided by the series for the average number of hours worked per man per quarter, #142 times the series on the number of persons employed, #147.

A third source of data is needed to estimate population, N. The data on population between the ages of 14 and 64 were obtained from the Department of Commerce, Bureau of the Census, Current Population Reports, Series P-25, 1970. Census figures are annual, and reported as of April first each year. To make this series quarterly, a linear growth path of population was assumed, and the values calculated with the following formula:

$$X_{ij} = X_{ij-1} + 1/4 (X_i - X_{i-1})$$

where i is the index for the year and takes the values of 1951, 52, ... 70, and where j is the quarter index and takes the values 2, 3, and 4. When $j = 1$, X_{ij} is the reported value in the year it represents.

Annual figures on the size of the labor force were obtained from the Federal Reserve Bulletin.

VIII. APPENDIX B

The full wage equation estimated for the first thirty-six quarters is as follows:

$$\ln W_t = 3.046 + .7691 \ln Y_t + .9988 \ln P_t + .4880 \ln N_t$$
$$ (1.338)\ (6.4168) \qquad\quad (4.171) \qquad\quad (.761)$$

$$+ .3985 \ln E_t$$
$$ (2.188)$$

$$\overline{R}^2 = .999$$

$$DW = 1.8660$$

$$\rho = .63$$

IX. APPENDIX C

To incorporate expectations into the wage equation, we begin with the wage Equation (5p), and modify it by replacing $\ln P$, by $\ln P^{**}$, which makes wages a function of expected prices, rather than actual prices:

(5t) $\quad \ln W = \dfrac{1}{\phi_1 + \gamma_1} \ln \rho_2 + \dfrac{1}{\phi_1 + \gamma_1} \ln Y_t$

$$+ \ln P^{**} - \frac{\{\gamma_2 + \gamma_3 \nu (1-\sigma)\}}{\phi_1 + \gamma_1} \ln N_{t+1} - \frac{\gamma_3 \sigma}{\phi_1 + \gamma_1} \ln E_t$$

We next define:

(5u) $\quad \ln P^{**}_t = \lambda \ln P_t + (1-\lambda) \ln P^{**}_{t-1}$

where $\ln P^{**}$ is seen as price expectations formulated as a decaying weighted average of past price levels which is condensed into the above expression by the application of the Koyck transformation on the decaying weighted average. This is, of course, a naive and only first approximation to price expectations for the wage equation. This formulation implicitly assumes in the coefficient of λ that the expectations of the suppliers and demanders of labor are identical. Further work, both theoretical and empirical are needed to build expectations properly into the wage equation. However as a first approximation, we built in expectations

using the previously discussed simplifications as follows:

$$(5t) \quad \ln W_t = \frac{1}{\phi_1+\gamma_1} \ln \rho_2 + \frac{1}{\phi_1+\gamma_1} \ln Y_t$$

$$+ \ln P_t^{**} - \frac{\{\gamma_2+\gamma_3\nu(1-\sigma)\}}{\phi_1+\gamma_1} \ln N_t + [1 - \frac{\gamma_3\sigma}{\phi_1+\gamma_1}] \ln E_t$$

where:

$$(5u) \quad \ln P_t^{**} = \lambda \ln P_t + (1-\lambda) \ln P_{t-1}^{**}$$

Then lagging wages by one period:

$$(5v) \quad \ln W_{t-1} = \frac{1}{\phi_1+\gamma_1} \ln \rho_2 + \frac{1}{\phi_1+\gamma_1} \ln Y_{t-1}$$

$$+ \ln P_{t-1}^{**} - \frac{\{\gamma_2+\gamma_3\nu(1-\sigma)\}}{\phi_1+\gamma_1} \ln N_{t-1}$$

$$+ [1 - \frac{\gamma_3\sigma}{\phi_1+\gamma_1}] \ln E_t$$

and solving for $\ln P_{t-1}^{**}$:

$$(5w) \quad \ln P_{t-1}^{**} = \ln W_{t-1} - \frac{1}{\phi_1+\gamma_1} \ln \rho_2 - \frac{1}{\phi_1+\gamma_1} \ln Y_{t-1}$$

$$+ \frac{\{\gamma_2+\gamma_3\nu(1-\sigma)\}}{\phi_1+\gamma_1} \ln N_{t-1} - [1 - \frac{\gamma_3\sigma}{\phi_1+\gamma_1}] \ln E_{t-1}$$

and subbing this into (5u):

(5x) $\ln P_t^{**} = \lambda \ln P_t + (1-\lambda)\ln W_{t-1} - \dfrac{(1-\lambda)}{\phi_1+\gamma_1} \ln \rho_2$

$$- \dfrac{(1-\lambda)}{\phi_1+\gamma_1} \ln Y_{t-1} + \dfrac{\{\gamma_2+\gamma_3\nu(1-\sigma)\}}{\phi_1+\gamma_1}(1-\gamma)\ln N_{t-1}$$

$$+ (1-\gamma)[1 - \dfrac{\gamma_3\sigma}{\phi_1+\gamma_1}]\ln E_{t-1}$$

and finally subbing this into (5t), the wage equation with expectations is derived:

(5y) $\ln W_t = \dfrac{\lambda}{\phi_1+\gamma_1} \ln\rho_2 + \dfrac{1}{\phi_1+\gamma_1} \ln Y_t + \lambda \ln P_t$

$$- \dfrac{\gamma_2+\gamma_3\nu(1-\sigma)}{\phi_1+\gamma_1}\} \ln N_t + [1 - \dfrac{\gamma_3\sigma}{\phi_1+\gamma_1}]\ln E_t$$

$$+ (1-\lambda)\ln W_{t-1} - \dfrac{(1-\lambda)}{\phi_1+\gamma_1} \ln Y_{t-1}$$

$$+ (1-\lambda) \dfrac{\{\gamma_2+\gamma_3\nu(1-\sigma)\}}{\phi_1+\gamma_1} \ln N_{t-1}$$

$$- (1-\lambda)[1- \dfrac{\gamma_3\sigma}{\phi_1+\gamma_1}]\ln E_{t-1}$$

$\ln W_t = .7141 + .7238 \ln Y_t + .5618 \ln P_t + .2145 \ln E_t$
   (1.04) (12.03) (6.224) (1.813)

$- .8565 \ln N_t - .3274 \ln Y_{t-1} + .1895 \ln E_{t-1}$
   (1.98) (3.88) (1.67)

$+ .5914 \ln N_{t-1} + .532 \ln W_{t-1}$
   (1.33) (7.78)

$\bar{R}^2 = .999$ $DW = 1.9$

$\rho = .97$

X. APPENDIX D

The table used to present the trade-offs in changes in the money supply to hold an endogenous variable constant, given a unit change in an exogenous variable, was computed in the following manner. Each coefficient in the table represents the negative of the ratio of reduced form coefficients, $-R_b/R_a$. R_b is the reduced form coefficient of the exogenous variable experiencing the unit change in the reduced form equation of the relevant endogenous variable whose constancy is required. R_a is the reduced form coefficient of the money supply in the reduced form equation of the relevant endogenous variable whose constancy is required.

For example, if we wish to compute the compensating change in the money supply required to keep prices constant given a unit change in productivity, we take the coefficient of productivity in the reduced form price equation, $-.0745$, as R_b. This is divided by the reduced form coefficient of the money supply in the price equation, $.2420$, or R_a. The negative of this ratio is then the first element in Table 4. The remaining elements are computed in a similar manner.